AMAT 3240 – Graph Theory **Assignment #6**
Fall 1999

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 8:00 am on Friday November 5, 1999.

1. Prove that a graph G is a tree if and only if it is connected and for every edge $e \in E(G)$, $G \setminus \{e\}$ is not connected.

   By way of definition, note that any edge e in a graph for which the deletion of e results in a disconnected graph (ie. a graph with at least 2 components), is called a *cut-edge*. So, the above statement can be rewritten as:

   Prove that a graph G is a tree if and only if it is connected and every edge is a cut-edge.

2. Suppose $\mathcal{T}$ is a tree with average degree a. Determine $|V(\mathcal{T})|$.

3. Suppose that G is a graph of order n such that for each $v \in V(G)$, $G \setminus \{v\}$ is a tree.

   (a) What is $|E(G)|$?

   (b) What is G?

4. Let G be a graph with weighted edges. Suppose that G has exactly one edge, say e_M, of maximum weight.

   (a) Find an example of a graph G on at least 5 vertices for which Kruskal's Algorithm always produces a spanning tree which includes e_M.

   (b) If G is a complete graph on at least 3 vertices, show that e_M cannot be in any minimum weight spanning tree.

5. Let G be a complete graph on 7 vertices, say $v_1, \ldots, v_7$. And let the edge $v_i v_j$ have weight $i + 2j + 3|i - j|$, where $i < j$.

   (a) Find a minimum weight spanning tree in G.

   (b) Find a minimum weight Hamiltonian cycle in G.