AMAT 3240 – Graph Theory **Assignment #5**
Fall 1999

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 8:00 am on Friday October 29, 1999.

1. Complete the proof of Theorem 10.11 of the textbook, by proving that if $\mathcal{T}$ is a transitive tournament, then each vertex of $\mathcal{T}$ has a different score.

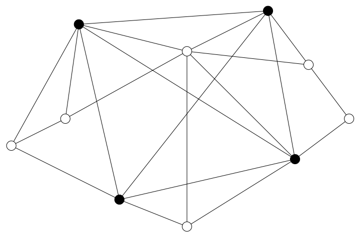
2. In a tournament, we define a *king* to be any vertex from which every vertex is reachable by a directed path of length at most 2.

   (a) Prove that every tournament must have at least one king.

   (b) Prove that every transitive tournament has exactly one king.

   (c) Is it true that a tournament is transitive if and only if it has exactly one king?

3. A *clique* is a subgraph which is isomorphic to a complete graph. For example, the following graph has a clique of order 4.



   Let $c(G)$ denote the order of a largest clique in a graph G. Prove that $c(G) = \alpha(\bar{G})$.

4. Prove that every tree with maximum degree $\Delta > 1$ has at least Δ vertices of degree 1.