AMAT 3240 – Graph Theory Fall 1999

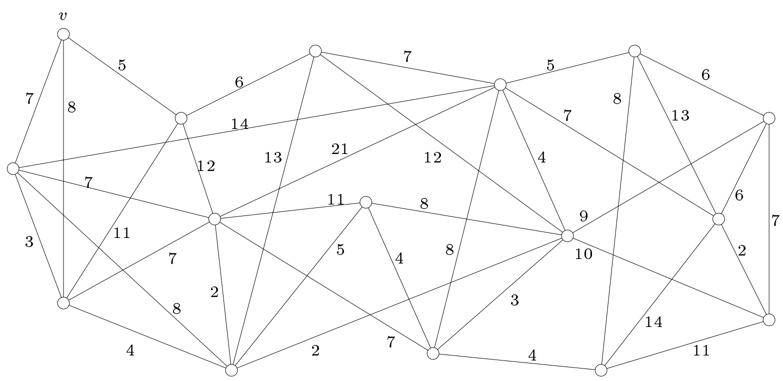
Assignment #4

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 8:00 am on October 15, 1999.

1. Let A denote an adjacency matrix for a graph G, such that $A_{i,j} = 1$ if and only if v_i and v_j are adjacent. Prove that $A^2_{i,j}$ (ie the entry in row i and column j of the matrix A^2) is the number of distinct walks of length 2 between v_i and v_j.

2. Find a least-weight path from v to each other vertex in the following graph:



3. Prove that a digraph D is Eulerian if and only if it is strongly connected and $\text{indeg}(v) = \text{outdeg}(v)$ for each vertex $v \in V(D)$.

4. Prove that if a digraph has a canonical ordering of its vertices then it is acyclic.

5. In the following digraph D, either find a canonical ordering of its vertices or else prove that no such ordering exists. Then find a least-weight directed path from v to each other vertex.