AMAT 3240 – Graph Theory **Assignment #3**

Fall 1999

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 8:00 am on October 5, 1999.

1. Prove that there do not exist any δ-regular graphs on n vertices, where δ and n are both odd.
2. Construct a graph which is
   (a) Hamiltonian but not Eulerian
   (b) Eulerian but not Hamiltonian
   (c) Eulerian and Hamiltonian
   (d) bipartite, Eulerian, and Hamiltonian
3. Let G be a connected graph with $|V(G)| \geq 3$ such that $\deg(u) + \deg(v) \geq k$ for each pair of non-adjacent vertices u and v, where $k < n$. Prove that G contains a path of length k.
4. There are 2^n binary vectors of length n. We can construct a graph by associating each of these 2^n vectors with a vertex, and joining two vertices by an edge if and only if their corresponding vectors differ in precisely one coordinate. This graph is called the binary n-cube, and is denoted by Q_n.
   (a) Draw Q_n for each of $n \in \{1, 2, 3, 4\}$.
   (b) For which n is Q_n Eulerian?
   (c) For which n is Q_n Hamiltonian?
5. A connected graph G has 12 vertices and 64 edges. Is G Hamiltonian? Is G Eulerian? Do we have enough information to be able to tell?
6. A subset S of $V(G)$ having the property that no pair of vertices in S is adjacent is called an *independent set* of vertices. Define the *independence number* of G, denoted by $\alpha(G)$, to be the maximum cardinality of the independent sets of vertices of G. Determine $\alpha(G)$ for:
   (a) K_n
   (b) $K_{m,n}$
   (c) the Petersen graph
7. A subset F of $E(G)$ having the property that no pair of edges in F is adjacent is called a *matching* in G. Let $\alpha'(G)$ to denote the size of a maximum matching of G. Determine $\alpha'(G)$ for:
   (a) K_n
   (b) $K_{m,n}$
   (c) the Petersen graph