AMAT 3240 – Graph Theory **Assignment #2**
Fall 1999

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 8:00 am on September 28, 1999.

1. Prove that if $G_1 \cong G_2$, then $|E(G_1)| = |E(G_2)|$. Does the converse also hold?

2. Define the relation $\sim$ as follows: $G_1 \sim G_2$ if there exists an edge $e \in E(G_1)$ such that $G_1 \setminus \{e\}$ is a subgraph of G_2.

   (a) Construct an example of three mutually non-isomorphic graphs (G_1, G_2, and H) such that $G_1 \sim H$ and $G_2 \sim H$.

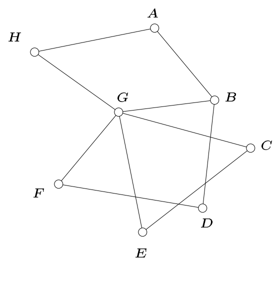
   (b) Is $\sim$ reflexive?

   (c) Is $\sim$ symmetric?

   (d) Is $\sim$ transitive?

   (e) Is $\sim$ an equivalence relation?

3. The function φ from the definition of isomorphism between two graphs is called an *isomorphism* function between the graphs. When dealing with a graph and itself, say G and G, we refer to φ as an *automorphism* of G (notice that an automorphism of G really is just a permutation of $V(G)$). One obvious automorphism for any given graph is the identity function. For the graph below, find all of its automorphisms.



4. Define $\text{dist}(u, v)$, the *distance* between vertices u and v, to be the length (ie. number of edges) of a shortest path between u and v. Define $\text{dia}(G)$, the *diameter* of G, to be the maximum distance between two vertices of G (ie. $\text{dia}(G) = \max\{\text{dist}(u, v) \mid u, v \in V(G)\}$). Define $\text{girth}(G)$, the *girth* of G, to be the length of a shortest cycle in G, or infinity if G is acyclic.

   Given a graph G, the complement $\bar{G}$ of G, is the graph having the same vertex set as G, but in which $uv \in E(\bar{G})$ if and only if $uv \notin E(G)$.

   Determine the diameter and girth of each of the following graphs, as well as their complements.

   (a) K_n, for $n = 1, 2, 3, \ldots$

   (b) $K_{m,n}$, for $m \leq n$ and $m = 1, 2, 3, \ldots$

   (c) the Petersen graph