

Instructions

- Answer each question completely; justify your answers.
  - This assignment is due at 4:00 pm on 22 February 2008.
1. Exercise 2.5.1, parts (c) and (d).
  2. Let  $A = \mathbb{N}$  and define  $\preceq$  on  $A$  by  $a \preceq b$  iff  $a$  divides  $b$ .
    - (a) Show that  $(A, \preceq)$  is a poset.
    - (b) Is the poset totally ordered?
    - (c) Does this poset have a maximum?
    - (d) Does this poset have a minimum?
  3. Let  $A = \{2, 3, 4, \dots, 17\}$  and define  $\preceq$  on  $A$  by  $a \preceq b$  iff  $a$  divides  $b$ .
    - (a) Draw the Hasse diagram for the poset  $(A, \preceq)$ .
    - (b) Is  $\preceq$  a total order?
    - (c) Does this poset have a maximum?
    - (d) Does this poset have a minimum?
    - (e) Does this poset have any minimal elements? If yes, what are they?
    - (f) Does this poset have any maximal elements? If yes, what are they?
    - (g) What is the least upper bound of elements 3 and 4?
    - (h) What is the greatest lower bound of elements 15 and 16?
  4. Let  $A = \mathbb{R}^2$  and define  $\preceq$  on  $A$  by  $(a, b) \preceq (x, y)$  iff  $a \leq x$  and  $b \leq y$ .
    - (a) Show that  $(A, \preceq)$  is a poset.
    - (b) Is the poset totally ordered?
    - (c) What is the least upper bound on  $(\sqrt{2}, 9)$  and  $(5, -3)$ ?
    - (d) What is the greatest lower bound on  $(\pi, \frac{2}{3})$  and  $(0, \frac{3}{2})$ ?
  5. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = \sqrt{x^3 - 4x}$ .
    - (a) State the domain of  $f$ .
    - (b) State the range of  $f$ .
    - (c) Prove or disprove:  $f$  is one-to-one.
    - (d) Prove or disprove:  $f$  is onto.
  6. Exercise 3.1.17.
  7. Exercise 3.1.19, except part (a).

8. Let  $A = \{0, 1, 2, \dots, 9\}$  and define the function  $g : \mathcal{P}(A) \rightarrow \mathbb{Z}$  so that  $g(X) = |X|$ .
- (a) What is the domain of  $g$ ?
  - (b) How many elements are in the domain of  $g$ ?
  - (c) What is the range of  $g$ ?
  - (d) Is  $g$  surjective?
  - (e) Is  $g$  injective?