

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday October 20th in Assignment Box #23.

1. Let $A = \{2, 3, 4, \dots, 17\}$ and define \preceq on A by $a \preceq b$ iff a divides b .
 - (a) Draw the Hasse diagram for the poset (A, \preceq) .
 - (b) Is \preceq a total order?
 - (c) Does this poset have a maximum?
 - (d) Does this poset have a minimum?
 - (e) Does this poset have any minimal elements? If yes, what are they?
 - (f) Does this poset have any maximal elements? If yes, what are they?
 - (g) What is the least upper bound of elements 3 and 4?
 - (h) What is the greatest lower bound of elements 15 and 16?
2. Let $A = \mathbb{R}^2$ and define \preceq on A by $(a, b) \preceq (x, y)$ iff $a \leq x$ and $b \leq y$.
 - (a) Show that (A, \preceq) is a poset.
 - (b) Is the poset totally ordered?
 - (c) What is the least upper bound on $(\sqrt{2}, 9)$ and $(5, -3)$?
 - (d) What is the greatest lower bound on $(\pi, \frac{2}{3})$ and $(0, \frac{3}{2})$?
3. Let $A = \{0, 1, 2, \dots, 7\}$ and define the function $g : \mathcal{P}(A) \rightarrow \mathbb{Z}$ so that $g(x) = |x|$.
 - (a) What is the domain of g ?
 - (b) How many elements are in the domain of g ?
 - (c) What is the range of g ?
 - (d) Is g surjective?
 - (e) Is g injective?
4. Define $h : \mathbb{N}^2 \rightarrow \mathbb{N}$ by $h : (x, y) \mapsto (x + y)$.
 - (a) Is h surjective?
 - (b) Is h injective?
5. Exercise 3.1.16.
6. Exercise 3.2.7, part (b).

Note that the part of the problem that asks an inverse to be found is asking for an inverse to the function $f : A \rightarrow (\text{Range of } f)$.