Name	**MUN Number**

Marks

[6] 1. State whether each statement is true, false, or invalid:

(a) If $x^2 \in \mathbb{R}$ then $x \in \mathbb{N}$.

(b) Let $a = 2k + 1$ for some $k \in \mathbb{Z}$.

(c) $A \Longrightarrow B$ if and only if $\text{not}(A) \Longrightarrow \text{not}(B)$.

(d) n is odd and $4 \mid n \Longleftrightarrow n^2 \leq 0$ or $2 = 3$.

(e) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Q}, x < y$.

(f) $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Q}, x < y$.

[6] 2. State the negation of each of the following:

(a) For all $a \in \mathbb{N}$ there exists $b \in \mathbb{Q}$ such that $a = -b$.

(b) There exists an integer n such that for every real number r, $n \mid r$.

[5] 3. Consider the statement: n is odd $\Longrightarrow n^2 + 2n + 1$ is even.

(a) What is the converse of this statement?

(b) Prove that this converse is true.

[7] 4. Let $A = \{2, 4, 7, 9\}$, $B = \{2, 4, 8\}$, and $C = \{3, 6, 9\}$.

(a) Draw a Venn diagram showing the relationship between the sets. Label each element.

(b) What are:

i. $A \cap B$

ii. $A \cup B$

iii. $(B \setminus A) \cap (A \cup C)$

iv. $A \setminus (B \cup C)$

v. $A \oplus B$

vi. $((A \cup C) \cap B)^2$

[5] 5. Let $A = \{a, \{a, b\}, c, d, \{d, e\}\}$.

(a) What is $|A|$?

(b) Indicate whether the following statements are true or false:

i. $a \in A$
ii. $b \in A$
iii. $b \subseteq A$
iv. $\emptyset \in A$
v. $\emptyset \subseteq A$
vi. $\{a, b\} \in A$
vii. $\{a, b\} \subseteq A$
viii. $\{\{a, b\}\} \subseteq A$
ix. $\mathcal{P}(\{c\}) \subseteq A$

[6] 6. Let A, B, and C be sets. Prove that $(A \setminus B) \times C = (A \times C) \setminus (B \times C)$.

[10] 7. Define $\sim$ on the set $A = \{-3, -2, -1, 0, 1, 2, 3\}$ by $x \sim y$ if and only if $x^2 + y^2$ is even.

(a) Is $\sim$ reflexive? Justify your answer.

(b) Is $\sim$ symmetric? Justify your answer.

(c) Is $\sim$ anti-symmetric? Justify your answer.

(this question continues...)

(d) Is $\sim$ transitive? Justify your answer.

(e) Is $\sim$ an equivalence relation? If yes, find both of $\overline{0}$ and $\overline{1}$.

[45]