

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 9:00 am on September 24, 2001.

The following symbols will be used to represent certain sets of numbers:

\mathbb{Z}	the set of integers
\mathbb{N}	the set of natural numbers, namely $\{1, 2, 3, \dots\}$
\mathbb{R}	the set of real numbers
\mathbb{Q}	the set of rational numbers

1. Determine whether the following statements are true, false, or invalid.
 - (a) If $4 \geq 4$ and 7 is odd, then $x^2 + 9x - 1$ has a real solution.
 - (b) Suppose n is a non-negative integer.
 - (c) If $-2^2 = 4$ then 3 is even and $5 < 4$.
 - (d) $\sqrt{x^2} = x$.
 - (e) 0 is positive.
2. For each valid statement in Question 1 that is an implication,
 - (a) state the converse
 - (b) determine whether the converse holds.
3. What is the negation of each of the following statements:
 - (a) A or $(B$ and not(C))
 - (b) A or B or C
 - (c) $(A$ and not(B)) and $(C$ or not(D))

Definition. For integers a and b , we say that a divides b (written as “ $a|b$ ”) if there exists an integer n such that $b = na$. An integer x is said to be even if $x = 2k$ for some integer k . And an integer x is said to be odd if $x = 2k + 1$ for some integer k .

4. Show that the following statements are false:
 - (a) Let $n \in \mathbb{N}$. Then $8|n^2$ if and only if $8|n$.
 - (b) If $x, y \in \mathbb{R}$ such that $x > 0$, $y > 0$, then $(x + 3)^2 + (y + 4)^2 \leq 5^2$.
 - (c) For all $x \in \mathbb{R}$, $100x^4 > \frac{x^6}{1000}$.

(over)

5. Rewrite the following statements as English sentences. Also indicate whether each statement is true or false.

(a) $\exists x \in \mathbb{Z}, \exists y \in \mathbb{Q}, x = y.$

(b) $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Q}, x = y.$

(c) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Q}, x = y.$

(d) $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Q}, x = y.$

6. Find the negation of each statement in Question 5 and indicate whether it is true or false.

7. Let $x, y \in \mathbb{Z}$. Prove that xy is odd if and only if x and y are both odd.

8. Consider the statement: $\forall x \in \mathbb{Z}, x \text{ is odd} \Rightarrow 4|(x^3 - x).$

(a) Is this statement true or false? Justify your answer either with a proof or else with a counter-example.

(b) What would be the negation of the statement?