TEST 2 — **PURE MATHEMATICS 2320** — FALL 2000

Name	**MUN Number**

Marks

[4] 1. Let $A = \mathbb{R} \setminus \{1\}$ and define $f : A \to A$ by $f : a \mapsto \dfrac{a}{a-1}$. Show that $f^{-1} = f$.

[7] 2. State whether each of the following sets is finite, countably infinite, or uncountable:

(a) $\mathbb{R}^2$

(b) $\emptyset$

(c) $7\mathbb{Z} + 3$

(d) $\{x \in \mathbb{R} \mid x^4 - 13x^2 + 10 = 0\}$

(e) $\{x \in \mathbb{Q} \mid x \in (-3, 4)\}$

(f) $\{x \in \mathbb{Z} \mid x = -\sqrt{x^2}\}$

(g) $\{x \in \mathbb{R}, \mid x = -\sqrt{x^2}\}$

[4] 3. (a) Express in base 10 the base 3 number 12011.

(b) Express in base 4 the base 10 number 234.

[5] 4. Let $a = 1728$ and $b = 804$.

(a) Calculate $\mathrm{GCD}(a, b)$.

(b) Find integers m and n such that $ma + nb = \mathrm{GCD}(a, b)$.

(c) Calculate $\mathrm{LCM}(a, b)$.

[3] 5. State whether each of the following is true or false:

(a) $84 \equiv 12 \pmod 4$

(b) If $x \not\equiv 0 \pmod 3$ then $x^{200} \equiv 1 \pmod 3$

(c) $-67 \equiv 17 \pmod{20}$

[6] 6. Solve for x:

(a) $2x \equiv 4 \pmod 8$

(b) $3x \equiv 0 \pmod{12}$

(c) $11x \equiv 4 \pmod{100}$

[4] 7. Prove that $3^{2n} - 1$ is divisible by 8 for every integer $n \geq 1$.

[6] 8. Define a sequence by $a_1 = 1$ and $a_k = 2a_{k-1} + 1$ for $k > 1$.

(a) What are the first 7 terms of this sequence?

(b) Find a formula for a_n and prove that it is correct.

[39]