

Name	MUN Number
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Marks

[3] 1. State whether each statement is true, false, or invalid:

(a) If $x \in \mathbb{Q}$ then $x \in \mathbb{Z}$.(b) Suppose a is a root of $f(x) = x^2 - 1$.(c) $A \implies B$ if and only if $\text{not}(B) \implies \text{not}(A)$.(d) 12 divides n or $3 = \sqrt{-9} \iff n$ is even and $2n + 1$ is even.(e) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Q}, x = y$.(f) $\mathbb{Q} = \left\{ \frac{a}{b} \mid a, b \in \mathbb{N} \right\}$.

[6] 2. State the negation of each of the following:

(a) For all $a \in A$ there exists $b \in B$ such that $a = b$.(b) There exists an integer n such that for every real number r , $n \mid r$.

- [7] 3. Let $A = \{1, 2, 3, 6\}$, $B = \{2, 4, 6\}$, and $C = \{3, 6, 9, 12\}$.
- (a) Draw a Venn diagram showing the relationship between the sets. Label each element.
- (b) What are:
- $A \cap C$
 - $A \cup B$
 - $A \cap (A \cup C)$
 - $A \setminus (B \cup C)$
 - $A \oplus B$
 - $((A \cup B) \cap C)^2$
- [4] 4. Let $A = \{a, b, \{a, b, c\}\}$.
- (a) What is $|A|$?
- (b) Indicate whether the following statements are true or false:
- $a \in A$
 - $c \in A$
 - $c \subseteq A$
 - $\emptyset \in A$
 - $\emptyset \subseteq A$
 - $\{a, b\} \in A$
 - $\{a, b\} \subseteq A$

[4] 5. Consider the statement: n is odd $\implies n^2$ is odd.

(a) What is the converse of this statement?

(b) Prove that this converse is true.

[6] 6. Define \sim on $\mathbb{Z} \setminus \{0\}$ by $x \sim y$ iff $xy > 0$.

(a) Is \sim reflexive?

(b) Is \sim symmetric?

(c) Is \sim anti-symmetric?

(d) Is \sim transitive?

(e) Is \sim an equivalence relation? If yes, what is $\bar{1}$?

[5] 7. Let T be a finite set.

(a) Show that $(\mathcal{P}(T), \subseteq)$ is a poset.

(b) i. Give an example of a non-empty finite set T for which $(\mathcal{P}(T), \subseteq)$ is totally ordered.

ii. Draw the Hasse diagram for your poset.

[5] 8. Define $f : \mathbb{R} \rightarrow \mathbb{R}$ by $f : a \mapsto \frac{a^2 + 6}{2}$.

(a) What is the domain of f ?

(b) What is the range of f ?

(c) Is f injective?

(d) Is f surjective?

(e) Is f bijective?

[2] 9. Define $f : \mathbb{Z} \rightarrow \mathbb{Z}$ by $f(x) = x^2 + x + 1$, and $g : \mathbb{Z} \rightarrow \mathbb{Z}$ by $g(x) = 2 - 5x$.
What is $(f \circ g)(x)$?