PMAT 2320 – Discrete Mathematics — Assignment #5
Fall 2000

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 9:00 am on October 20, 2000.

1. Let $A = \mathbb{N}$ and define $\preceq$ on A by $a \preceq b$ iff a divides b.
   (a) Show that $(A, \preceq)$ is a poset.
   (b) Is the poset totally ordered?
   (c) Does this poset have a maximum?
   (d) Does this poset have a minimum?

2. Let $A = \{2, 3, 4, \ldots, 15\}$ and define $\preceq$ on A by $a \preceq b$ iff a divides b.
   (a) Draw the Hasse diagram for this poset.
   (b) Does this poset have a maximum?
   (c) Does this poset have a minimum?
   (d) Does this poset have any minimal elements? If yes, what are they?
   (e) Does this poset have any maximal elements? If yes, what are they?

3. Let $A = \mathbb{R}^2$ and define $\preceq$ on A by $(a, b) \preceq (x, y)$ iff $a \leq x$ and $b \leq y$.
   (a) Show that $(A, \preceq)$ is a poset.
   (b) Is the poset totally ordered?
   (c) What is the least upper bound on $(\pi, 7)$ and $(5, -1)$?
   (d) What is the greatest lower bound on $(9, \frac{12}{17})$ and $(0, \frac{9}{7})$?

4. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \sqrt{x^2 + 1}$.
   (a) State the domain of f.
   (b) State the range of f.
   (c) Prove or disprove: f is one-to-one.
   (d) Prove or disprove: f is onto.