**PMAT 2320 – Discrete Mathematics** **Assignment #4**
**Fall 2000**

**Instructions**

- Answer each question completely; justify your answers.
- This assignment is due at 9:00 am on October 13, 2000.

1. For each binary relation $\mathcal{R}$ determine whether $\mathcal{R}$ is reflexive, symmetric, and/or transitive. If $\mathcal{R}$ is an equivalence relation, then also determine $\overline{1}$.

   (a) $\mathcal{R} = \{(1,5), (1,8), (5,8), (8,1)\}$
   (b) $\mathcal{R} = \{(x,y) \in \mathbb{Z}^2 \mid x^2 - y^2 \text{ is even}\}$
   (c) $\mathcal{R} = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 5\}$
   (d) $\mathcal{R} = \{(x,y) \in \mathbb{Z}^2 \mid xy > 0\}$

2. Define $\sim$ on $\mathbb{Z}$ by $x \sim y$ if $2x + y = 3n$ for some integer $n$.

   (a) Show that $\sim$ is an equivalence relation on $\mathbb{Z}$.
   (b) What is $\overline{0}$?
   (c) What is $\overline{1}$?
   (d) What is $\overline{2}$?
   (e) What is $\mathbb{Z}/\!\sim$?

3. Define $\sim$ on $\mathbb{R}^2$ by $(a,b) \sim (x,y)$ if $a - b = x - y$.

   (a) Show that $\sim$ is an equivalence relation on $\mathbb{R}^2$.
   (b) What is the equivalence class of $(3,4)$? Also describe $\overline{(3,4)}$ geometrically.

4. Let $T = \{1,2,3,4,5\}$. Define $\sim$ on $\mathcal{P}(T)$ by $A \sim B$ if $A \subseteq B$.

   (a) Is $\sim$ reflexive?
   (b) Is $\sim$ symmetric?
   (c) Is $\sim$ antisymmetric?
   (d) Is $\sim$ transitive?