PMAT 2320 – Discrete Mathematics — Assignment #2
Fall 2000

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 9:00 am on September 29, 2000.

1. Rewrite the following statements as English sentences. Also indicate whether each statement is true or false.

   (a) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Q},\ x = y$.

   (b) $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Q},\ x = y$.

   (c) $\exists x \in \mathbb{Z}, \exists y \in \mathbb{Q},\ x = y$.

   (d) $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Q},\ x = y$.

2. Find the negation of each statement in Question 1 and indicate whether it is true or false.

3. Consider the statement: $\forall x \in \mathbb{Z}$, x is odd $\Rightarrow 8 \mid (x^2 - 1)$.

   (a) Is this statement true or false? Justify your answer either with a proof or else with a counter-example.

   (b) What would be the negation of the statement?

4. Let $A = \{1, 2, 3, 4\}$, $B = \{3, 6, 9, 12\}$, and $C = \{2, 4, 6, 8\}$.

   (a) Draw a Venn diagram showing the relationship between the sets. Label each element.

   (b) What are:

      i. $A \cap B$

      ii. $B \cup C$

      iii. $A \cap (B \cup C)$

      iv. $(A \cap B) \cup C$

      v. $A \setminus (B \cup C)$

      vi. $(B \cup C) \setminus A$

5. Let $A = \{a, b, c, \{a, b, c, d\}, \{c, d, e\}, f, \{g, h\}\}$.

   (a) What is $|A|$?

   (b) Indicate whether the following statements are true or false:

      i. $d \in A$

      ii. $e \in A$

      iii. $f \in A$

      iv. $\emptyset \in A$

      v. $\emptyset \subseteq A$

      vi. $d \subseteq A$

      vii. $\{a, b, c\} \subseteq A$

      viii. $\{a, b, c\} \in A$

      ix. $\{a, b, c, d\} \subseteq A$

      x. $\{a, b, c, d\} \in A$

      xi. $\{c, f, \{g, h\}\} \subseteq A$