PMAT 2320 – Discrete Mathematics **Assignment #1**
Fall 2000

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 9:00 am on September 22, 2000.

The following symbols will be used to represent certain sets of numbers:

$\mathbb{Z}$	the set of integers
$\mathbb{N}$	the set of natural numbers, namely $\{1, 2, 3, \dots\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{Q}$	the set of rational numbers

1. Determine whether the following statements are true, false, or invalid. For each which is a valid implication, also determine whether the converse holds.

   (a) If 6 is even then 25 is a perfect square.

   (b) If $x^2 = 4$ then $x = 2$.

   (c) $\sqrt{x^2} = x$.

   (d) Let a be a root of $f(x) = x^2 - 9$.

   (e) If $0 = -1$ then 10 is odd or $1 + 1 \neq 2$.

   (f) $8 \leq 8$.

2. Let $a, b, c \in \mathbb{Z}$. Prove that if $a|b$ and $a|c$, then $a|(b + c)$. Does the converse also hold?

3. Let $x, y \in \mathbb{Z}$. Prove that xy is odd if and only if x and y are both odd.

4. What is the negation of each of the following statements:

   (a) A and (B or not(C))

   (b) A and B and C

   (c) (A or not(B)) or (C and not(D))

5. Show that the following statements are false:

   (a) Let $n \in \mathbb{N}$. Then $4|n^2$ if and only if $4|n$.

   (b) If $x, y \in \mathbb{R}$ such that $x > 0$, $y > 0$, and $x \neq 2y$, then $\dfrac{2x - y}{2y - x} \geq 0$.

   (c) If $x \in \mathbb{R}$ such that $x \notin \{-3, -4\}$, then $\dfrac{x}{x + 3} < \dfrac{x + 2}{x + 4}$.