MATH 4341 – Combinatorial Design Theory — Assignment #6
Winter 2020

1. Recall that a design $(V, \mathcal{B})$ is called *resolvable* if its blocks can be partitioned into sets that partition V. Prove that no resolvable $(v, \frac{v}{2}, \frac{v}{2} - 1)$-BIBD can exist when $v \equiv 2 \pmod 4$.

2. A Steiner triple system that is resolvable is called a *Kirkman Triple System*. Prove that if a KTS(v) exists then $v \equiv 3 \pmod 6$.

3. Let $1 < m < n$ be integers. A Latin square L of order n has a *subsquare* of order m if there is an $m \times m$ subarray of L that is itself a Latin square on a subset of m of the n symbols of L. Prove that if a Latin square of order n has a subsquare of order m then $2m \leqslant n$.

4. (a) Construct a symmetric idempotent quasigroup of order 9 on the set $X = \mathbb{Z}_9$.
   (b) When this quasigroup is used to construct a STS(27) via the Bose construction, what are the blocks that contain the point 3_0?
   (c) When this quasigroup is used to construct a STS(27) via the Bose construction, what are the blocks that contain the point 6_1?

5. (a) Construct a symmetric half-idempotent quasigroup of order 8 on the set $X = \mathbb{Z}_8$.
   (b) When this quasigroup is used to construct a STS(25) via the Skolem construction, what are the blocks that contain the point 3_0?
   (c) When this quasigroup is used to construct a STS(25) via the Skolem construction, what are the blocks that contain the point 6_1?