MATH 4341 – Combinatorial Design Theory Assignment #5
Winter 2020

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at noon on Thursday March 19th.

1. Recall that Heffter's First Difference Problem is to find a partition of $\{1, 2, \ldots, 3m\}$ into 3-subsets of the form $\{a, b, c\}$ such that $a + b = c$ or $a + b + c \equiv 0 \pmod{6m + 1}$.

   (a) Find a solution to Heffter's First Difference Problem for $m = 3$ and then use it to construct a STS(19).

   (b) Find a solution to Heffter's First Difference Problem for $m = 4$ and then use it to construct a STS(25).

2. Find a Skolem triple system of order 8 and use it to construct a STS.

3. Find an O'Keefe triple system of order 10 and use it to construct a STS.

4. Show that if there exists a Rosa triple system of order m involving a partition of the set $\{1, 2, \ldots, 3m + 2\} \setminus \{2m + 1, 3m + 1\}$ then $m \equiv 1$ or $2 \pmod{4}$.

5. Construct a cyclic STS(57).

6. Construct a cyclic STS(69).