MATH 4341 – Combinatorial Design Theory Assignment #4
Winter 2020

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at noon on Thursday February 27th.

1. Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D\}$.
   Let $\mathcal{A} = \{1234, 1567, 189A, 1BCD, 258B, 269C, 27AD, 359D, 36AB, 378C, 45AC, 468D, 479B\}$.
   Let $\mathcal{B} = \{1234, 1567, 189A, 1BCD, 258B, 269C, 27AD, 35AC, 368D, 379B, 459D, 46AB, 478C\}$.

   (a) Randomly select a block A_0 (*i.e.,* actually pick one) from $\mathcal{A}$ and also a block B_0 from $\mathcal{B}$ and construct the residual designs $Res(X, \mathcal{A}, A_0)$ and $Res(X, \mathcal{B}, B_0)$.

   (b) Prove that the two designs constructed in part (a) are isomorphic.

   (c) Find an isomorphism between $(X, \mathcal{A})$ and $(X, \mathcal{B})$.

2. Find a $(15, 7, 3)$-difference set in $(\mathbb{Z}_{15}, +)$.

3. Verify that $\{0, 1, 7, 19, 23, 44, 47, 49\}$ is a $(57, 8, 1)$-difference set in $(\mathbb{Z}_{57}, +)$.

4. Find a $(31, 15, 7)$-difference set in $(\mathbb{Z}_{31}, +)$.

5. (a) Find a $(21, 5, 1)$ difference set.

   (b) Use it to construct a projective plane of order 4.

6. Construct a $(31, 5, 2)$ difference family.

7. Let L and L^* be two Latin squares of order n, and let $L_{i,j}$ (resp. $L^*_{i,j}$) denote the symbol contained in the cell whose location is row i column j of L (resp. L^*). The squares L and L^* are said to be *orthogonal* if the set of ordered pairs $\{(L_{i,j}, L^*_{i,j}) : 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant n\}$ has cardinality n^2 (i.e., each possible ordered pair of symbols occurs, and it does so exactly once). A set $\{L_1, L_2, \ldots, L_t\}$ of t Latin squares of order n is said to be a set of *mutually orthogonal Latin squares* if each pair of Latin squares is orthogonal.

   (a) Find a pair of orthogonal Latin squares of order 3.

   (b) Find a set of three mutually orthogonal Latin squares of order 4.

   (c) Show that there is no pair of orthogonal Latin squares of order 2.