MATH 4341 – Combinatorial Design Theory Assignment #1
Winter 2020

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at the start of class on Wednesday January 22nd.

1. Show that for a $(v, 3, 1)$-BIBD to exist it is necessary that $v \equiv 1$ or $3 \pmod 6$.

2. What are the necessary conditions for the existence of a $(v, 4, 1)$-BIBD?

3. Prove that every (6,3,2)-BIBD is simple.

4. (a) Consider the set $S = \{1, 2, 3, 4, 5, 6\}$. Find a partition of S into ordered subsets of size three, such that each subset is either of the form $\{x, y, z\}$ such that $x + y = z$ or of the form $\{x, y, z\}$ such that $x + y + z = 13$.

   (b) For each set $A = \{x, y, z\}$ from part (a), where $x < y < z$, let A' be the set $\{0, x, x + y\}$. Let σ be the permutation $(0\ 1\ 2 \cdots 12)$. For each set A', list the sets $\sigma^i(A')$ for $i = 0, 1, 2, \ldots, 12$.

   (c) Taken as blocks, what type of design do the sets from part (b) form?

5. Let A_0 be a block in a $(v, k, 1)$-BIBD, say $(X, \mathcal{A})$.

   (a) Find a formula for the number of blocks $A \in \mathcal{A}$ such that $|A \cap A_0| = 1$.

   (b) Use this formula to show that $b \geqslant k(r - 1) + 1$ if a $(v, k, 1)$-BIBD exists.

   (c) Using the facts that $vr = bk$ and $v = r(k - 1) + 1$, deduce that $(r - k)(r - 1)(k - 1) \geqslant 0$ and hence $r \geqslant k$, which implies Fisher's Inequality.

6. Let A_0 be a block in a $(v, k, 1)$-BIBD, say $(X, \mathcal{A})$. Let $x \in X \setminus A_0$ and show that there are at least k blocks that contain x and intersect A_0. From this, deduce that $r \geqslant k$, which implies Fisher's Inequality.