MATH 4341 – Combinatorial Design Theory Assignment #6
Winter 2018

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 12:00 on Monday April 2nd in Assignment Box #32.

1. Use the SQS doubling construction to build a SQS(8).

2. Let $(X, \mathcal{A})$ be a t-(v, k, λ) design and let $\mathcal{B} = \{X \setminus A : A \in \mathcal{A}\}$. Prove that $(X, \mathcal{B})$ is also a t-design and determine its parameters in terms of v, k and λ.

3. (a) Construct a symmetric idempotent quasigroup of order 9 on the set $X = \mathbb{Z}_9$.
   (b) When this quasigroup is used to construct a STS(27) via the Bose construction, what are the blocks that contain the point 3_0?
   (c) When this quasigroup is used to construct a STS(27) via the Bose construction, what are the blocks that contain the point 6_1?

4. (a) Construct a symmetric half-idempotent quasigroup of order 8 on the set $X = \mathbb{Z}_8$.
   (b) When this quasigroup is used to construct a STS(25) via the Skolem construction, what are the blocks that contain the point 3_0?
   (c) When this quasigroup is used to construct a STS(25) via the Skolem construction, what are the blocks that contain the point 6_1?

5. (a) Describe how to construct an idempotent quasigroup of every even order $n \geqslant 4$.
   (b) Describe how to use the construction from part (a) to construct a TTS($3n$).
   (c) Use these constructions to build a TTS(12).

6. (a) Let $1 < m < n$ be integers. A Latin square L of order n has a *subsquare* of order m if there is an $m \times m$ subarray of L that is itself a Latin square on a subset of m of the n symbols of L. Prove that if a Latin square of order n has a subsquare of order m then $2m \leqslant n$.
   (b) Let $1 < m < n$ be integers. Suppose that $L_1, L_2, \ldots, L_s$ are MOLS of order n. Suppose further that $L_1, L_2, \ldots, L_s$ each have a subsquare of order m situated in the same positions (without loss of generality, in the upper left $m \times m$ corner). Prove that $(s+1)m \leqslant n$ and also that these subsquares form a set of s MOLS of order m.

7. A Steiner triple system that is resolvable is called a *Kirkman Triple System*. Prove that if a KTS(v) exists then $v \equiv 3 \pmod 6$.