MATH 4341 – Combinatorial Design Theory Assignment #5
Winter 2018

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday March 15th in Assignment Box #32.

1. Show that if there exists a Rosa triple system of order m involving a partition of the set $\{1, 2, \ldots, 3m+2\} \setminus \{2m+1, 3m+1\}$ then $m \equiv 1$ or $2 \pmod 4$.

2. Construct a cyclic STS(57).

3. Construct a cyclic STS(69).

4. Suppose $\mathcal{S}$ is a Steiner triple system of order n. Make three copies of $\mathcal{S}$, one on the point set $X_1 = \{1, 2, \ldots, n\}$, a second on $X_2 = \{n+1, n+2, \ldots, 2n\}$ and the third on $X_3 = \{2n+1, 2n+2, \ldots, 3n\}$. Suppose that L_1, L_2 and L_3 are three mutually orthogonal Latin squares of order n, each of them on the symbol set $\{1, 2, \ldots, n\}$. For $t \in \{1, 2, 3\}$, add $(t-1)n$ to each cell of L_t, yielding a Latin square L_t^*. Now construct n^2 triples $\{L_1^*(i,j), L_2^*(i,j), L_3^*(i,j)\}$ where $1 \leqslant i \leqslant n$, $1 \leqslant j \leqslant n$ and $L_t^*(i,j)$ denotes the symbol contained in the cell whose location is row i column j of L_t^*. Verify that these n^2 triples, when combined with those of the three copies of $\mathcal{S}$, form a STS$(3n)$.

5. Assuming that a 5-$(12, 6, 1)$ design exists, determine λ_i for each $i \in \{0, 1, \ldots, 4\}$.

6. Prove that if an $S(2, k, v)$ exists then $v - 1 \geqslant k(k-1)$. Moreover, prove that if inequality occurs then in fact $v \geqslant k^2$.

7. Show that if an $S(t, k, v)$ exists then $v - t + 1 \geqslant (k - t + 1)(k - t + 2)$. As a corollary, show that the existence of an $S(5, 8, v)$ requires that $v \geqslant 24$.