# MATH 4341 – Combinatorial Design Theory  Assignment #4
# Winter 2018

**Instructions**

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday March 8th in Assignment Box #32.

1. (a) Find a $(21, 5, 1)$ difference set.

   (b) Use it to construct a projective plane of order 4.

2. Construct a $(31, 5, 2)$ difference family.

3. Recall that Heffter's First Difference Problem is to find a partition of $\{1, 2, \ldots, 3m\}$ into 3-subsets of the form $\{a, b, c\}$ such that $a + b = c$ or $a + b + c \equiv 0 \pmod{6m + 1}$.

   (a) Find a solution to Heffter's First Difference Problem for $m = 3$ and then use it to construct a STS(19).

   (b) Find a solution to Heffter's First Difference Problem for $m = 4$ and then use it to construct a STS(25).

4. Find a Skolem triple system of order 8 and use it to construct a STS.

5. Find an O'Keefe triple system of order 10 and use it to construct a STS.

6. Let $L$ and $L^*$ be two Latin squares of order $n$, and let $L_{i,j}$ (resp. $L^*_{i,j}$) denote the symbol contained in the cell whose location is row $i$ column $j$ of $L$ (resp. $L^*$). The squares $L$ and $L^*$ are said to be *orthogonal* if the set of ordered pairs $\{(L_{i,j}, L^*_{i,j}) : 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant n\}$ has cardinality $n^2$ (i.e., each possible ordered pair of symbols occurs, and it does so exactly once). A set $\{L_1, L_2, \ldots, L_t\}$ of $t$ Latin squares of order $n$ is said to be a set of *mutually orthogonal Latin squares* if each pair of Latin squares is orthogonal.

   (a) Find a pair of orthogonal Latin squares of order 3.

   (b) Find a set of three mutually orthogonal Latin squares of order 4.

   (c) Show that there is no pair of orthogonal Latin squares of order 2.

7. Let $v \equiv 2 \pmod 4$. Prove that no resolvable $(v, \frac{v}{2}, \frac{v}{2} - 1)$-BIBD can exist.