MATH 4341 – Combinatorial Design Theory Assignment #2
Winter 2018

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday February 1st in Assignment Box #32.

1. Suppose that A is a block of a (v, k, λ)-BIBD. For each $i \in \{0, 1, \ldots, k\}$, let x_i denote the number of blocks other than A that intersect A in precisely i elements. Prove that $\sum_{i=0}^{k} ix_i = k(r-1)$ and also that $\sum_{i=0}^{k} i(i-1)x_i = k(k-1)(\lambda-1)$.

2. Suppose $(X, \mathcal{A})$ is a PBD such that $v = |X|$, $\lambda = 1$ and the blocks of $\mathcal{A}$ have size 3 and k. Prove that if $v \equiv 2 \pmod 3$ then $k \equiv 2 \pmod 3$.

3. Suppose that a (v, k, λ)-BIBD is both a derived design and a residual design (of some other design(s)). Prove that $v = 2\lambda + 2$.

4. Let $(X, \mathcal{A})$ be a symmetric (v, k, λ)-BIBD. Prove that $k > \lambda$.

5. Let $(X, \mathcal{A})$ be a symmetric $(v, k, 1)$-BIBD and let A_0 be any block of $\mathcal{A}$. Explain how the blocks of the residual design $Res(X, \mathcal{A}, A_0)$ can be partitioned into resolution classes.

6. Another way of defining a projective plane is as a set X of points and a set $\mathcal{A}$ of subsets of X called lines such that the following three axioms hold:

   A1: Given any two points, there is exactly one line that contains both of them.

   A2: Given any two lines, there is exactly one point that is contained in both of them.

   A3: There is a subset of X consisting of 4 points, no three of which are collinear.

   Prove that under these axioms, every line of the design $(X, \mathcal{A})$ contains the same number of points.