MATH 4341 – Combinatorial Design Theory   Assignment #2
Winter 2015

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday January 29th in Assignment Box #37.

1. Suppose that a BIBD has $v = 8$ and $k = 4$. Prove that $b \geqslant 14$.

2. Suppose $(X, \mathcal{A})$ is a PBD such that $v = |X|$, $\lambda = 1$ and the blocks of $\mathcal{A}$ have size 3 and k. Prove that if $v \equiv 2 \pmod{3}$ then $k \equiv 2 \pmod{3}$.

3. Let $(X, \mathcal{A})$ be a symmetric (v, b, r, k, λ)-BIBD and let A_0 be any block of $\mathcal{A}$. Prove that $\left(X \setminus A_0, \{A \setminus A_0 : A \in \mathcal{A} \setminus \{A_0\}\}\right)$ is a BIBD and determine its five parameters.

4. Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D\}$.
   Let $\mathcal{A} = \{1234, 1567, 189A, 1BCD, 258B, 269C, 27AD, 359D, 36AB, 378C, 45AC, 468D, 479B\}$.
   Let $\mathcal{B} = \{1234, 1567, 189A, 1BCD, 258B, 269C, 27AD, 35AC, 368D, 379B, 459D, 46AB, 478C\}$.

   (a) Select a block A_0 (*i.e.*, actually pick one) from $\mathcal{A}$ and also a block B_0 from $\mathcal{B}$ and use the construction from question 3 to construct two new designs by deleting the chosen blocks from the two given designs $(X, \mathcal{A})$ and $(X, \mathcal{B})$.

   (b) Prove that the two designs constructed in part (a) are isomorphic.

   (c) Find an isomorphism between $(X, \mathcal{A})$ and $(X, \mathcal{B})$.