MATH 4341 – Combinatorial Design Theory Assignment #7
Winter 2013

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 15:00 on Friday March 15th in Assignment Box #48.

1. Prove: if there is a $T(\lambda, v)$ then $(\lambda, 6) = 1$ implies $v \equiv 1$ or $3 \pmod 6$, $(\lambda, 6) = 2$ implies $v \equiv 0$ or $1 \pmod 3$, and $(\lambda, 6) = 3$ implies $v \equiv 1 \pmod 2$.

2. If n is odd, define $L_n = (\ell_{ij})$ by $\ell_{ij} = 2i - j \pmod n$. If n is even and $n > 2$, define T_n to be the Latin square of order n derived from L_{n-1} by replacing each of $\ell_{1,2}, \ell_{2,3}, \dots, \ell_{n-2,n-1}, \ell_{n-1,n}, \ell_{n,1}$ by n and then appending a new row and column that make the square Latin. Prove that L_n and T_n are idempotent Latin squares.

3. Suppose L is a symmetric Latin square of side n and that the symbol x occurs k times on the main diagonal.

   (a) How many times does x occur above the diagonal?

   (b) Prove that $k \equiv n \pmod 2$.

   (c) If L is also idempotent, prove that n is odd.

4. Use a Latin square of side 4 to construct a STS(13).

5. Use a Latin square of side 5 to construct a STS(15).