MATH 4341 – Combinatorial Design Theory Assignment #5
Winter 2013

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 15:00 on Friday February 15th in Assignment Box #48.

1. Suppose A is a binary matrix. Prove that A is the incidence matrix of a regular block design if and only if $J_v A = kJ_{v\times b}$ and $AJ_b = rJ_{v\times b}$ for some integers v, b, r, k.

2. Prove that if $t, v, k \in \mathbb{N}$ such that $t \leqslant k \leqslant v$ then there is a t-(v, k, λ) design with $\lambda = \binom{v-t}{k-t}$.

3. Prove that the set of all k-subsets of a v-set is a t-design provided that $t \leqslant k \leqslant v$.

4. Suppose that $\mathcal{D}$ is a t-design and that x is a treatment in S, where S is the support set of $\mathcal{D}$. Let $\mathcal{D}^x$ be the design with support set $S \setminus \{x\}$ and whose blocks are the blocks of $\mathcal{D}$ that do not contain x. Prove that $\mathcal{D}^x$ is a $(t-1)$-design.

5. Find an affine plane $AG(2, 3)$.

6. Prove that every BIBD$(n^2, n^2 + n, n + 1, n, 1)$ is an affine plane.