MATH 4340 – Combinatorial Analysis — Assignment #5
Fall 2013

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 15:00 on Friday November 1st in Assignment Box #34.

1. Find a generating function for the number of ways to withdraw $\$r$ from an ATM that dispenses all denominations of paper currency that are in circulation. In how many ways can \$1000 be withdrawn?

2. Find $[x^6]\ G(x)$, where

   (a) $G(x) = (1-x)^{-4}$.

   (b) $G(x) = (1-x^2)^{-1}$.

   (c) $G(x) = (1+x)^{-3}$.

3. Use a generating function to determine the number of integers x, $0 \leqslant x \leqslant 999999$, whose digits sum to 25.

4. Find $[x^r]\ G(x)$, where $G(x) = \left(\sum_{k\geqslant 0} k^2 x^k\right)\left(\sum_{k\geqslant 0}(k+1)x^k\right)$.

5. Suppose that Nestlé has agreed to provide an airline with small packages of Smarties that are to be distributed to the airline's passengers while in flight. Each package is to contain 12 individual Smarties, coloured red, blue, green, and/or yellow. Colours red, green, and yellow each need to be represented at least thrice per package, but blue should occur at most twice (thereby attempting to design the blue candies to be rare).

   (a) How many different ways are there to fill each package, subject to these constraints?

   (b) What fraction of the possible ways to fill each package contain at least one blue Smartie?

6. For each integer $r \geqslant 0$, find $[x^r]\ G(x)$, where $G(x) = \left(\frac{1}{1-x^2}\right)\left(\frac{2}{1-x^4}\right)$.

7. Let a_r denote the number of ways to express r as a sum of perfect squares of positive integers.

   (a) Find a generating function for a_r.

   (b) Determine a_{31}.

8. Find a generating function for a_r, the number of integer solutions to the equation $e_1 + 3e_2 + 5e_3 + 7e_4 = r$, where $0 \leqslant e_1$, $0 \leqslant e_2$, $2 \leqslant e_3 \leqslant 8$, and $0 \leqslant e_4 \leqslant 10$.

9. Find a generating function for a_r, the number of partitions of r into four parts.

10. Exercise 6.3.14.