MATH 4340 – Combinatorial Analysis — Assignment #3
Fall 2013

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 15:00 on Friday October 11th in Assignment Box #34.

1. Exercise 5.3.34.
2. In how many ways can Premier Dunderdale assign 15 different cabinet portfolios to 33 MHAs
   (a) without restriction?
   (b) such that no MHA receives more than one portfolio?
3. How many nonnegative integer solutions are there to: $6 \leqslant x_1 + x_2 + x_3 < 12$?
4. Exercise 5.4.28.
5. How many nonnegative integer solutions are there to the system of equations: $\sum_{i=1}^{8} x_i = 44$ and $x_1 + x_3 + x_6 = 12$?
6. Exercise 5.4.48.
7. Fifteen differently coloured subway cars are to be configured into three trains, each of which will be put into service on a different subway line. In how many ways can these three trains be configured, such that
   (a) each train has at least one car?
   (b) each train has at least three cars
   (c) each train has at least five cars

   In each case, assume that train is oriented so that there is no question of which car is the first car and which is the last car.
8. Evaluate $\sum_{k=0}^{n} 6k^3 - 6k$.
9. Evaluate $\sum_{k=0}^{n+1} k\binom{n}{n-k+1}$.
10. Prove via a committee-selection argument: $\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$.
11. Prove via a block-walking argument: $\sum_{k=0}^{m} \binom{m}{k}\binom{n}{r+k} = \binom{m+n}{m+r}$.