

Instructions

- Answer each question completely; justify your answers.
1. Solve for x by using the Index Calculus Method:
 - (a) $55^x \equiv 444 \pmod{569}$
 - (b) $7^x \equiv 92 \pmod{1433}$
 2. Show that the Diffie-Hellman Problem (DHP) and the El Gamal Problem (ELGAMAL) are computationally equivalent.
 3. Suppose that Alice has published the key $(1237, 34, 383)$ for use in the El Gamal public-key cryptosystem.
 - (a) You wish to send the message $m = 14$ to Alice. What do you actually transmit?
 - (b) You have monitored the transmission $(94, 225)$ to Alice.
 - i. Use the Index Calculus Method for solving the Discrete Log Problem to find Alice's secret key a .
 - ii. What was the original message?
 4. Prove that each of the following numbers is composite by finding a Fermat witness:
 - (a) 123
 - (b) 52687
 - (c) 1263739430742009409841
 5. Show that 52633 is composite and also that it has no Fermat witnesses.
 6. Show that 75361 is composite and also that it has no Fermat witnesses.