

Instructions

- Answer each question completely; justify your answers.
 - This assignment is due at 23:59 (Newfoundland time) on Tuesday March 9th.
 - Submit your assignment via the D2L shell for the course.
1. Find integers q and r with $0 \leq r < |b|$ such that $a = qb + r$:
 - (a) $a = 239, b = 7$
 - (b) $a = -8641, b = 6$
 - (c) $a = -9162, b = -27$
 - (d) $a = 27153, b = -35$
 2. Let $a = 6408$ and $b = 1296$. Let g be the greatest common divisor of a and b .
 - (a) Find g .
 - (b) Find integers m and n such that $ma + nb = g$.
 - (c) What is the least common multiple of a and b ?
 3. Let $a = 84876$ and $b = 13794$. Let g be the greatest common divisor of a and b .
 - (a) Find g .
 - (b) Find integers m and n such that $ma + nb = g$.
 - (c) What is the least common multiple of a and b ?
 4. Let a and b be integers that are not both zero. Prove that $\gcd(a, a + b) = \gcd(a, b)$.
 5. Find integers x and y such that $158x + 264y = 4$.
 6. Show that there is no integer solution to $196x + 245y = 8$.
 7. Prove: if $k \in \mathbb{N}$ then $\gcd(3k + 2, 5k + 3) = 1$.
 8. Find the prime decompositions for:
 - (a) $n = 1701513$
 - (b) $n = 2476500$
 9. Let $a, b \in \mathbb{Z}$ and let p be a prime. Are the following assertions true or false? Justify your answers.
 - (a) If $p \mid (a + b)$ and $p \mid (a - b)$ then $p \mid a$.
 - (b) If $p \mid a$ and $p \mid (a^2 + b^2)$ then $p \mid b$.
 - (c) If $p \mid (a^9 + a^{17})$ then $p \mid a$.
 10. Suppose a, b, p are integers such that p is prime, $\gcd(a, p^2) = p$ and $\gcd(b, p^3) = p^2$. Prove that $\gcd(a + b, p^4) = p$.