MATH 2320 – Discrete Mathematics — Assignment #4
Winter 2021

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 23:59 (Newfoundland time) on Tuesday February 16th.
- Submit your assignment via the D2L shell for the course.

1. Define the relation $\sim$ on $A = \mathbb{Z}$ by $a \sim b$ if $a^2 = b^2$.
   (a) Prove that $\sim$ is an equivalence relation.
   (b) What is $\overline{1}$?
   (c) What is $\overline{0}$?
   (d) What is $A/\!\sim$?

2. Let $A = \mathbb{N}$ and define the relation $\preceq$ on A by $a \preceq b$ if $a^3 \leqslant b^3$.
   (a) Show that $(A, \preceq)$ is a poset.
   (b) Is the poset totally ordered?
   (c) Does this poset have a maximum? If yes, what is it?
   (d) Does this poset have a minimum? If yes, what is it?

3. Let $A = \{2, 3, 4, \ldots, 20\}$ and define $\preceq$ on A by $a \preceq b$ if a divides b.
   (a) Draw the Hasse diagram for the poset $(A, \preceq)$.
   (b) Is $\preceq$ a total order?
   (c) Does this poset have a maximum? If yes, what is it?
   (d) Does this poset have a minimum? If yes, what is it?
   (e) Does this poset have any minimal elements? If yes, what are they?
   (f) Does this poset have any maximal elements? If yes, what are they?
   (g) What is the least upper bound of elements 6 and 8?
   (h) What is the greatest lower bound of elements 6 and 8?

4. Let $A = \mathbb{R}^2$ and define $\preceq$ on A by $(a, b) \preceq (x, y)$ if $a \leqslant x$ and $b \leqslant y$.
   (a) Show that $(A, \preceq)$ is a poset.
   (b) Is the poset totally ordered?
   (c) What is the least upper bound on $(\sqrt{2}, -7)$ and $(5, 6)$?
   (d) What is the greatest lower bound on $(\pi, \frac{3}{4})$ and $(0, \frac{4}{3})$?

(over)

5. Let $A = \{1, 2, 3, \ldots, 10\}$ and define the function $g : \mathcal{P}(A) \to \mathbb{Z}$ so that $g(x) = |x|$.

   (a) What is the domain of g?

   (b) How many elements are in the domain of g?

   (c) What is the range of g?

   (d) Is g surjective?

   (e) Is g injective?

   (f) Is g bijective?

6. Define $h : \mathbb{N}^2 \to \mathbb{N}$ by $h : (x, y) \mapsto (x + y)$.

   (a) State the range of h.

   (b) Is h surjective?

   (c) Is h injective?

   (d) Is h bijective?

7. Let $f : \mathbb{N} \to \mathbb{Q}$ be defined by $f(x) = \frac{x-2}{x+1}$.

   (a) Is h surjective?

   (b) Is h injective?

8. An integer that is greater than 1 is called *prime* if its only positive divisors are 1 and itself. In Chapter 4 we will prove that every integer n that exceeds 1 can be uniquely expressed as a product of primes.

   Let $A = \mathbb{N} \setminus \{1\}$. Define the function $f : A \to A$ such that $f(n)$ is the smallest prime divisor of n.

   (a) Is f surjective?

   (b) Is f injective?

   (c) Define the relation $\sim$ on A by $a \sim b$ if $f(a) = f(b)$.
   Is $\sim$ an equivalence relation? If so, what is $\overline{30}$?