MATH 2320 – Discrete Mathematics   Assignment #3
Winter 2021

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 23:59 (Newfoundland time) on Tuesday February 9th.
- Submit your assignment via the D2L shell for the course.

1. Prove or disprove: for all sets A, B and C, $\left(A \cap B = A \cap C\right) \Rightarrow \left(B = C\right)$.

2. Prove or disprove: for all sets A, B and C, $\left(A \oplus B = A \oplus C\right) \Rightarrow \left(B = C\right)$.

3. Let A and B be sets. Prove: $(A \cap B)^c = A^c \cup B^c$.

4. Determine whether the relation $\mathcal{R}$ is reflexive:

   (a) $\mathcal{R} = \{(x, y) \in \mathbb{Z}^2 \mid x^2 - y^2 \text{ is even}\}$
   (b) $\mathcal{R} = \{(x, y) \in \mathbb{Q}^2 \mid x^2 y \geqslant 0\}$

5. Determine whether the relation $\mathcal{R}$ is symmetric:

   (a) $\mathcal{R} = \{(x, y) \in \mathbb{N}^2 \mid x^2 + y > 0\}$
   (b) $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x - y = 0\}$

6. Determine whether the relation $\mathcal{R}$ is antisymmetric:

   (a) $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x \leqslant y\}$
   (b) $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x^2 \geqslant y^2\}$

7. Determine whether the relation $\mathcal{R}$ is transitive:

   (a) $\mathcal{R} = \{(x, y) \in \mathbb{Z}^2 \mid x - y \text{ is odd}\}$
   (b) $\mathcal{R} = \{(x, y) \in \mathbb{Z}^2 \mid xy \text{ is even}\}$

8. Define the relation $\sim$ on $A = \mathbb{Z}$ by $a \sim b$ if $|a| = |b|$.

   (a) Prove that $\sim$ is an equivalence relation.
   (b) What is $\overline{1}$?
   (c) What is $\overline{0}$?
   (d) What is $A/\!\sim$?

9. Define the relation $\sim$ on $A = \mathbb{R}^2$ by $(a, b) \sim (x, y)$ if $a^2 + b = x^2 + y$.

   (a) Prove that $\sim$ is an equivalence relation.
   (b) Provide a geometric description of $\overline{(1, -1)}$.