MATH 2320 – Discrete Mathematics   Assignment #2
Winter 2021

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 23:59 (Newfoundland time) on Tuesday February 2nd.
- Submit your assignment via the D2L shell for the course.

1. Determine whether the following statement is a tautology, a contradiction, or neither:
$$\big((P \text{ and } (\text{not } Q)) \Rightarrow Q\big) \Leftrightarrow \big((P \text{ and } (\text{not } Q)) \Rightarrow (\text{not } P)\big)$$

2. Is the statement $P \Rightarrow (Q \text{ or } R)$ logically equivalent to the statement $(P \text{ and } (\text{not } Q)) \Rightarrow R$?
   Explain why or why not.

3. Let $A = \{w, x, y, z\}$. List all of the subsets B of A such that

   (a) $\{x, y, z\} \subseteq B$

   (b) $\{x, y, z\} \nsubseteq B$

   (c) $\{x, y, z\} \subset B$

   (d) $B \subseteq \{x, y, z\}$

   (e) $B \nsubseteq \{x, y, z\}$

   (f) $B \not\subset \{x, y, z\}$

4. Let $A = \{1, 2, 3, 9\}$, $B = \{2, 3, 5, 6, 7, 8, 9\}$, and $C = \{2, 4, 8\}$.

   (a) Draw a Venn diagram showing the relationship between the sets, and where each element belongs.

   (b) What are:

      i. $A \cap B$

      ii. $B \cup C$

      iii. $A \cup (B \cap C)$

      iv. $(A \cup B) \cap C$

      v. $A \setminus (B \cap C)$

      vi. $(A \setminus B) \cap C$

      vii. $A \oplus B$

      viii. $(B \cup C) \setminus A$

      ix. $(B \oplus C) \setminus A$

      x. $\mathcal{P}(C)$

      xi. $\mathcal{P}(B \cap C)$

      xii. $(A \cap C) \times B$

5. Let $A = \{a, b, c, \{a, b\}, \{a, b, c, d, e\}, f, \{e, f, g, h\}\}$.

   (a) What is $|A|$?

   (b) Indicate whether the following statements are true or false:

      i. $\emptyset \in A$
      ii. $f \in A$
      iii. $g \in A$
      iv. $\{f, g\} \in A$
      v. $\{f, g\} \subseteq A$
      vi. $\emptyset \subseteq A$
      vii. $f \subseteq A$
      viii. $\{a, b, c\} \subseteq A$
      ix. $\{a, b, c\} \in A$
      x. $\{b, f\} \subseteq A$
      xi. $\{b, f\} \in A$
      xii. $\{a, b\} \subseteq A$
      xiii. $\{a, b\} \in A$

6. Let $A = (-\infty, 5)$, $B = [-7, 14)$, $C = (2, 10]$, and $U = \mathbb{R}$. What are:

   (a) $A \cap B$
   (b) $B \cup C$
   (c) $A^c \setminus (B \cap C)$
   (d) $(A \cup C) \setminus (A \cup B)^c$
   (e) $B \oplus C$
   (f) $C \setminus B^c$

7. Let A, B and C be subsets of some universal set U.
   Prove: $A \setminus (B \setminus C) = (A \setminus B) \cup (A \setminus C^c)$.

8. Consider the statement: $\forall$ sets A, B, C and D, $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$.
   Is this statement true? If yes, prove it; otherwise show that it is false.

9. Consider the statement: $\exists$ sets A, B, C and D such that $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$.
   Is this statement true? If yes, prove it; otherwise show that it is false.