MATH 2320 – Discrete Mathematics Assignment #1
Winter 2021

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 23:59 (Newfoundland time) on Tuesday January 26th.
- Submit your assignment via the D2L shell for the course.

The following symbols will be used to represent certain sets of numbers:

$\mathbb{N}$	the set of natural numbers, namely $\{1, 2, 3, \ldots\}$
$\mathbb{Z}$	the set of integers
$\mathbb{Q}$	the set of rational numbers
$\mathbb{R}$	the set of real numbers
$\mathbb{C}$	the set of complex numbers

1. State whether the following are true, false, or not assertions.
   (a) (9 is odd and $5 \geqslant 5$) $\Rightarrow -1^2 = 1$
   (b) $\exists x \in \mathbb{R}$ such that $x^2 - 3x + 1 > 0$
   (c) $\forall x \in \mathbb{R}$, $x^2 - 3x + 1 > 0$
   (d) 0 is even
   (e) Let n be a non-negative integer.
   (f) If $x \in \mathbb{Z}$ then $x = \sqrt{x^2}$
2. For each assertion in Question 1 that is an implication,
   (a) state the converse of the implication
   (b) determine whether the converse holds
3. State the negation of each of the following statements (assuming that A, B and C are themselves statements with truth values):
   (a) A and (B or not(C))
   (b) (A or not B) or C
   (c) ((not(A)) and (B)) or (C or not(D))

Definition. For integers a and b, we say that "a divides b" (written as "$a \mid b$") if there exists an integer q such that $b = qa$. Otherwise a does not divide b and we can write "$a \nmid b$".

4. Explain why each of the following statements is false:
   (a) $6 \mid 32$
   (b) $\forall n \in \mathbb{N}$, $16 \mid n^2$ if and only if $8 \mid n$
   (c) If $x, y \in \mathbb{R}$ such that $x > 0$ and $y > 0$, then $(x + 5)^2 + (y + 12)^2 \leqslant 13^2$

(over)

5. Rewrite the following statements as English sentences. Also indicate whether each statement is true or false.

   (a) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Q}$ such that $x < y$.

   (b) $\exists x \in \mathbb{Z}$ such that $\forall y \in \mathbb{Q}$, $x \leqslant y$.

6. Find the negation of each statement in Question 5 and indicate whether it is true or false.

7. Let $x, y \in \mathbb{Z}$. Prove that xy is even if and only if x is even or y is even.

8. Consider the statement: $\forall x \in \mathbb{Z}$, x is odd $\Rightarrow 4 \mid (7x + x^3)$.
   Is this statement true or false?
   Justify your answer either with a proof or else with a counter-example.

9. Let a, b, u and v be integers such that $u \neq 0$ and $v \neq 0$.
   Consider the statement P: If $au + bv = 0$ then $a = b = 0$.

   (a) Is P true? If yes, then prove P; otherwise show that P is false.

   (b) State the converse of P.

   (c) State the contrapositive of P.

   (d) State the negation of P.

10. Suppose that $a_1, a_2, \ldots, a_7$ are positive integers and let m be their product.
    Prove that at least one of $a_1, a_2, \ldots, a_7$ is at least $\sqrt[7]{m}$.