

1. Prove: for each integer $n \geq 1$, $n^3 + 5n$ is divisible by 6.
2. Let $n \in \mathbb{N}$. Prove that $\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$.
3. Suppose the sum of the first eight terms of an arithmetic sequence is thrice the sum of the first five terms, and, moreover, the sum of the first ten terms is 1475. Find a formula for the sequence and calculate the sum of the first 50 terms.
4. If the second term of a geometric sequence is 8 and the seventh term is $-\frac{1}{4}$, find the sum of the first 100 terms.
5. Find $1 + \frac{3}{4} + \frac{7}{16} + \frac{15}{64} + \frac{31}{256} + \dots$
6. Let $a_0 = 5$, $a_1 = 17$, and for each $n \geq 1$ define $a_{n+1} = -8a_n - 16a_{n-1}$. Find a formula for a_n .
7. Let $a_0 = 3$, $a_1 = 2$, and for each $n \geq 1$ define $a_{n+1} = 20a_{n-1} - a_n$. Find a formula for a_n .
8. Consider the sequence defined by $a_0 = 2$, $a_1 = 3$ and for each $n \geq 2$, $a_n = -a_{n-1} - a_{n-2}$. Determine a_n in general, and then use your solution to determine a_3 .