MATH 2320 – Discrete Mathematics **Assignment #8**
Winter 2020

1. Suppose a, b, p are integers such that p is prime, $\gcd(a, p^2) = p$ and $\gcd(b, p^3) = p^2$. Prove that $\gcd(a + b, p^4) = p$.

2. Suppose p and $p + 2$ are both prime and that $p > 3$. Prove that $6 \mid (p + 1)$.

3. Find the prime decompositions for:

   (a) $n = 1701513$

   (b) $n = 2476500$

4. Reduce a modulo n

   (a) $a = 456723$, $n = 21$

   (b) $a = (9675)(5679)$, $n = 43$

   (c) $a = (901)^{68}(651)^{200}$, $n = 12$

5. Solve the following congruences:

   (a) $3x \equiv 19 \pmod{47}$

   (b) $7x \equiv 18 \pmod{431}$

   (c) $8x \equiv 9 \pmod{21}$

   (d) $8x \equiv 16 \pmod{28}$

   (e) $6x \equiv 0 \pmod{18}$

   (f) $7x \equiv 18 \pmod{43102}$

   (g) $3x \equiv 19 \pmod{48}$

6. Solve the following systems of congruences:

   (a) $5x - 2y \equiv 0 \pmod{11}$ and $2x + y \equiv 8 \pmod{11}$

   (b) $8x + 4y \equiv 2 \pmod{23}$ and $x - 3y \equiv 7 \pmod{23}$

   (c) $6x - 7y \equiv 8 \pmod{33}$ and $4x + 2y \equiv 3 \pmod{33}$

7. Solve the following system of congruences:
$$\begin{aligned} x &\equiv 42 \pmod{77} \\ x &\equiv 77 \pmod{100} \end{aligned}$$

8. Solve the following system of congruences:
$$\begin{aligned} x &\equiv 7 \pmod{12} \\ x &\equiv 5 \pmod{25} \\ x &\equiv 9 \pmod{37} \end{aligned}$$

9. Solve the following system of congruences:
$$\begin{aligned} x &\equiv 3 \pmod{4} \\ x &\equiv 2 \pmod{9} \\ x &\equiv 4 \pmod{25} \\ x &\equiv 7 \pmod{49} \end{aligned}$$