

1. Explain why $|\mathbb{Z}^2| = \aleph_0$
2.
 - (a) Give an example of an infinite collection of distinct finite sets whose intersection is finite.
 - (b) Give an example of an infinite collection of distinct infinite sets whose intersection is finite.
 - (c) Give an example of an infinite collection of distinct infinite sets whose intersection is infinite.
3. Find integers q and r with $0 \leq r < |b|$ such that $a = qb + r$:
 - (a) $a = 139, b = 6$
 - (b) $a = -8241, b = 5$
 - (c) $a = -9362, b = -17$
 - (d) $a = 27133, b = -45$
4. Let $a = 8850$ and $b = 975$. Let g be the greatest common divisor of a and b .
 - (a) Find g .
 - (b) Find integers m and n such that $ma + nb = g$.
 - (c) What is the least common multiple of a and b ?
5. Let $a = 876$ and $b = 1915$. Let g be the greatest common divisor of a and b .
 - (a) Find g .
 - (b) Find integers m and n such that $ma + nb = g$.
 - (c) What is the least common multiple of a and b ?
6. Let $a = 81876$ and $b = 13191$. Let g be the greatest common divisor of a and b .
 - (a) Find g .
 - (b) Find integers m and n such that $ma + nb = g$.
 - (c) What is the least common multiple of a and b ?
7. Let a and b be integers that are not both zero. Prove that $\gcd(a, a + b) = \gcd(a, b)$.
8. Find integers x and y such that $154x + 260y = 4$.
9. Show that there is no integer solution to $196x + 245y = 3$.
10. Prove: if $k \in \mathbb{N}$ then $\gcd(3k + 2, 5k + 3) = 1$.