MATH 2320 – Discrete Mathematics Assignment #5
Winter 2020

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday March 12th in Assignment Box #43.

1. Let $A = \mathbb{Z}^2$ and for $a = (a_1, a_2)$ and $b = (b_1, b_2)$ in A define $a \preceq b$ if $a_1 \leqslant b_1$ and $a_1 + a_2 \leqslant b_1 + b_2$.

   (a) Prove that $\preceq$ is a partial order on A.

   (b) Is $\preceq$ a total order on A? Justify your answer with a proof or a counterexample.

2. Let $A = \{1, 2, 3, 4, 5, 6\}$ and define the function $g : \mathcal{P}(A) \to \mathbb{Z}$ so that $g(x) = |x|$.

   (a) What is the domain of g?

   (b) How many elements are in the domain of g?

   (c) What is the range of g?

   (d) Is g surjective?

   (e) Is g injective?

   (f) Is g bijective?

3. Define $h : \mathbb{N}^2 \to \mathbb{N}$ by $h : (x, y) \mapsto x + y$.

   (a) State the range of h.

   (b) Is h surjective?

   (c) Is h injective?

   (d) Is h bijective?

4. Let $f : \mathbb{N} \to \mathbb{Q}$ be defined by $f(x) = \frac{x-2}{x+1}$.

   (a) Is h surjective?

   (b) Is h injective?

5. Let $S = \{1, 2, 3, 4\}$ and $T = \{a, b, c, d\}$. Define functions $f : S \to T$ and $g : S \to S$ such that $f = \{(1, d), (2, b), (3, c), (4, a)\}$ and $g = \{(1, 3), (2, 4), (3, 1), (4, 4)\}$.

   (a) Either find $f \circ f$ or explain why it does not exist.

   (b) Either find $f \circ g$ or explain why it does not exist.

   (c) Either find $g \circ f$ or explain why it does not exist.

   (d) Either find $g \circ g$ or explain why it does not exist.

   (e) Is f surjective? Is f injective? Find f^{-1} if it exists.

   (f) Is g surjective? Is g injective? Find g^{-1} if it exists.

(over)

6. Let $A = \{x \in \mathbb{R} \,|\, x \neq \frac{1}{3}\}$ and define $f : A \to \mathbb{R}$ by $f(x) = \frac{x}{3x-1}$.

   (a) Show that f is injective.

   (b) Is f is surjective?

   (c) What is the range of f?

   (d) Let D be the range of f and define $g : A \to D$ such that $g : x \mapsto f(x)$. What is g^{-1}?

7. Let $f : A \to B$ and $g : B \to C$ be functions. Prove that if $g \circ f$ is injective then f is injective.