MATH 2320 – Discrete Mathematics Winter 2020

Assignment #4

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday February 27th in Assignment Box #43.
- 1. Prove or disprove: for all sets A, B and C, $A \cap B = A \cap C \Rightarrow B = C$.
- 2. Prove or disprove: for all sets A, B and C, $A \oplus B = A \oplus C \Rightarrow B = C$.
- 3. Let A, B and C be sets. Prove: $(A \cap B) \times C = (A \times C) \cap (B \times C)$.
- 4. Define the relation \sim on $A = \mathbb{Z}$ by $a \sim b$ if |a| = |b|.
 - (a) Prove that \sim is an equivalence relation.
 - (b) What is $\overline{8}$?
 - (c) What is $\overline{0}$?
 - (d) What is A/\sim ?

5. Define the relation \sim on $A = \mathbb{R}^2$ by $(a, b) \sim (c, d)$ if 2a + b = 2c + d.

- (a) Prove that \sim is an equivalence relation.
- (b) Provide a geometric description of (5, 3).
- 6. Define the relation \sim on $A = \mathbb{Z}$ by $a \sim b$ if 3a + b is even.
 - (a) Prove that \sim is an equivalence relation.
 - (b) What is $\overline{3}$?
 - (c) What is A/\sim ?
- 7. Let $A = \mathbb{N}$ and define \preceq on A by $a \preceq b$ if $a^3 \leq b^3$.
 - (a) Show that (A, \preceq) is a poset.
 - (b) Is the poset totally ordered?
 - (c) Does this poset have a maximum? If yes, what is it?
 - (d) Does this poset have a minimum? If yes, what is it?
- 8. Let $A = \{2, 3, 4, \dots, 16\}$ and define \leq on A by $a \leq b$ if a divides b.
 - (a) Draw the Hasse diagram for the poset (A, \preceq) .
 - (b) Is \leq a total order?
 - (c) Does this poset have a maximum? If yes, what is it?
 - (d) Does this poset have a minimum? If yes, what is it?
 - (e) Does this poset have any minimal elements? If yes, what are they?
 - (f) Does this poset have any maximal elements? If yes, what are they?
 - (g) What is the least upper bound of elements 3 and 4?
 - (h) What is the greatest lower bound of elements 15 and 16?