MATH 2320 – Discrete Mathematics Assignment #3
Winter 2020

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday February 13th in Assignment Box #43.

1. Let A and B be sets. Prove: $(A \cap B)^c = A^c \cup B^c$.
2. Consider the statement: $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$ for all sets A, B, C and D. Is this statement true? If yes, prove it; otherwise show that it is false.
3. Let $A = (-\infty, -6)$, $B = [-8, -\pi)$, $C = (-4, \sqrt{2}]$, and $U = \mathbb{R}$. What are:
   (a) $A \cap B$
   (b) $B \cup C$
   (c) $A^c \setminus (B \cap C)$
   (d) $(A \cup C) \setminus (A \cup B)^c$
   (e) $B \oplus C$
   (f) $C \setminus B^c$
4. Let A, B and C be subsets of some universal set U. Prove: $A \setminus (B \setminus C) = (A \setminus B) \cup (A \setminus C^c)$.
5. Determine whether the relation $\mathcal{R}$ is reflexive:
   (a) $\mathcal{R} = \{(x, y) \in \mathbb{Z}^2 \mid x^2 + y^2 \text{ is even}\}$
   (b) $\mathcal{R} = \{(x, y) \in \mathbb{Q}^2 \mid xy > 0\}$
6. Determine whether the relation $\mathcal{R}$ is symmetric:
   (a) $\mathcal{R} = \{(x, y) \in \mathbb{N}^2 \mid x + y = 1000\}$
   (b) $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x - y < 49\}$
7. Determine whether the relation $\mathcal{R}$ is antisymmetric:
   (a) $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x \leqslant y\}$
   (b) $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x^2 \leqslant y^2\}$
8. Determine whether the relation $\mathcal{R}$ is transitive:
   (a) $\mathcal{R} = \{(x, y) \in \mathbb{Z}^2 \mid x + y \text{ is odd}\}$
   (b) $\mathcal{R} = \{(x, y) \in \mathbb{Z}^2 \mid xy \text{ is odd}\}$