MATH 2320 – Discrete Mathematics Assignment #2
Winter 2020

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday February 6th in Assignment Box #43.

1. Let a, b, u and v be integers such that $u \neq 0$ and $v \neq 0$.
   Consider the statement P: If $au + bv = 0$ then $a = b = 0$.
   (a) Is P true? If yes, then prove P; otherwise show that P is false.
   (b) State the contrapositive of P.
   (c) State the converse of P.
   (d) State the negation of P.

2. Let a_1, a_2, a_3 be positive integers and let $m = \prod_{i=1}^{3} a_i$.
   Prove that at least one of a_1, a_2, a_3 is at least $\sqrt[3]{m}$.

3. Prove that $\log_2 5$ is irrational.

4. Determine whether the following statement is a tautology: P or $\big((P \text{ and } (\text{not } Q)) \Rightarrow R\big)$

5. Is the statement $P \Rightarrow (Q \text{ or } R)$ logically equivalent to the statement $(P \text{ and } (\text{not } Q)) \Rightarrow R$? Explain why or why not.

6. Let $A = \{w, x, y, z\}$. List all of the subsets B of A such that
   (a) $\{x, y\} \subseteq B$
   (b) $\{x, y\} \not\subseteq B$
   (c) $\{x, y\} \subset B$
   (d) $B \subseteq \{x, y\}$
   (e) $B \not\subseteq \{x, y\}$
   (f) $B \not\subset \{x, y\}$

7. Let $A = \{1, 4, 8, 9\}$, $B = \{3, 4, 6, 7, 9\}$, and $C = \{2, 4, 6\}$.
   (a) Draw a Venn diagram showing the relationship between the sets, and where each element belongs.
   (b) What are:
      i. $A \cap B$
      ii. $B \cup C$
      iii. $A \cup (B \cap C)$
      iv. $(A \cup B) \cap C$
      v. $A \setminus (B \cap C)$
      vi. $(A \setminus B) \cap C$
      vii. $(B \cup C) \setminus A$
      viii. $\mathcal{P}(C)$

(over)

8. Let $A = \big\{a, b, \{a, b, c\}, \{a, c, d, e, g\}, f, \{e, f, g\}\big\}$.

(a) What is $|A|$?

(b) Indicate whether the following statements are true or false:

i. $\emptyset \in A$

ii. $f \in A$

iii. $g \in A$

iv. $\{f, g\} \in A$

v. $\{f, g\} \subseteq A$

vi. $\emptyset \subseteq A$

vii. $f \subseteq A$

viii. $\{a, b, c\} \subseteq A$

ix. $\{a, b, c\} \in A$

x. $\{b, f\} \subseteq A$

xi. $\{b, f\} \in A$

xii. $\{a, g\} \subseteq A$

xiii. $\{a, g\} \in A$