

1. There are 10 people participating in a discussion that employs a talking stick to designate who may speak.
  - (a) In how many ways can the 10 participants each speak once?
  - (b) In how many ways can half of the participants speak once each?
2. Let  $X$  and  $Y$  be finite sets with  $|X| = m$  and  $|Y| = n$ .
  - (a) How many bijective functions  $f : X \rightarrow X$  are there?
  - (b) How many injective functions  $f : X \rightarrow Y$  are there?
3. In the Lotto 6/49 lottery, a player buys a ticket by selecting six numbers from the set  $\{1, 2, \dots, 49\}$ , in the hope of matching the six numbers later chosen at random by the lottery commission.
  - (a) How many possible tickets are there?
  - (b) What is the probability of winning the lottery jackpot?
  - (c) What is the probability that the six winning numbers are consecutive?
  - (d) What is the probability that the six winning numbers are all prime?
4. In a class of 25 students, the teacher assigns teams of 5 students to work together on group projects. Each team is given a distinct project.
  - (a) In how many ways can the teams be formed?
  - (b) In how many ways can the teams be formed, with the additional requirement that each team must have a spokesperson to give an oral report?
5. A committee consisting of six politicians is to be formed from among 50 US governors and 10 Canadian premiers.
  - (a) In how many ways can the committee be formed?
  - (b) In how many ways can the committee be formed with equal representation from each country?
  - (c) If the committee is formed randomly, what is the probability of unequal representation happening?
  - (d) If the committee is formed randomly, what is the probability of there being a Canadian majority?
6. A fair coin is tossed seven times. What is the probability of:
  - (a) all heads?
  - (b) exactly three heads?
  - (c) at most two heads?
  - (d) an odd number of heads?
  - (e) an even number of heads?

(over)

7. Repeat Question #6 but with a biased coin that favours heads with probability  $\frac{3}{5}$ .
8. A basket contains five red balls, four white balls, and three blue balls. Two balls are drawn in sequence, with the first ball being replaced before the second one is drawn. What is the probability of drawing:
- (a) two white balls?
  - (b) at most one white ball?
  - (c) at least one white ball?
  - (d) one red and one white ball?
  - (e) no white balls?
9. Repeat Question #8 but without replacing the first ball before drawing the second one.
10. A florist sells 6 different colours of roses. In how many ways can a dozen roses be bought:
- (a) without restriction?
  - (b) so that each colour occurs at least once?
  - (c) so that exactly two colours are selected?
11. Prove:  $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$
12. Expand and simplify:  $(2x^3 - x^2)^8$
13. Expand and simplify:  $\left(3x^2 - \frac{2}{x^3}\right)^5$
14. Find the coefficient of the  $x^{16}$  term of  $(2x^3 - x)^{25}$ .  
Also find the coefficient of the  $x^{33}$  term.
15. Find the coefficient of the  $x^{48}$  term of  $\left(6x^2 + \frac{1}{x^2}\right)^{20}$ .  
Also find the constant term.