MATH 2320 – Discrete Mathematics **Assignment #10**
Winter 2020

1. In a group of 91 students, 73 are taking Math, 42 are taking English, and 7 are taking neither. How many students are taking both English and Math?

2. When a group of people were surveyed about whether they like the coffee at three local stores known as Alice's, Bob's and Charlie's, 626 said they liked Alice's, 452 liked Bob's, and 440 liked Charlie's. Some of these people liked more than one store's coffee: 109 liked both Alice's and Bob's, 51 liked both Alice's and Charlie's, and 311 liked both Bob's and Charlie's. If 9 people like all three coffee stores, how many people were surveyed?

3. Suppose U is a set containing 75 elements and that A_1, A_2, A_3 and A_4 are subsets of U such that each subset A_i contains 28 elements, the intersection of any two of the four subsets contains 12 elements, the intersection of any three of the four subsets contains 5 elements, and the intersection of all four subsets contains 1 element.

   (a) What is $|A_1 \cup A_2 \cup A_3 \cup A_4|$?

   (b) How many elements belong to exactly two of the four subsets?

4. Find the number of integers between 1 and 10000 inclusive that are:

   (a) divisible by at least one of 3, 5, 7, 11.

   (b) divisible by 3 and 5, but not by 7 or 11.

   (c) divisible by exactly three of 3, 5, 7, 11.

5. In how many ways can two adjacent squares be selected from an 8×8 chessboard?

6. How many three-digit numbers contain the digits 2 and 5 but none of 0, 3, 7?

7. How many possible licence plates can be manufactured if they must show three letters followed by three numeric digits...

   (a) without any further restriction?

   (b) if the digits must be distinct?

   (c) if the letters must be distinct?

   (d) if the digits and letters must all be distinct?

8. George has promised to increase his vocabulary by learning 90 new words during the summer. Suppose he has 53 days in which to accomplish this task and that he will learn at least one new word on each of these days. Show that during some span of consecutive days he will learn exactly 15 new words.

9. Of any five points chosen within an equilateral triangle whose sides have length 1, show that at least two are within a distance of $\frac{1}{2}$ of each other.

10. Let $S = \{1, 2, 3, \ldots, 200\}$ and let $A \subseteq S$ such that $|A| = 101$. Show that A contains two distinct elements x and y such that $x \mid y$.