

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday January 23rd in Assignment Box #??????.

The following symbols will be used to represent certain sets of numbers:

- \mathbb{N} the set of natural numbers, namely $\{1, 2, 3, \dots\}$
- \mathbb{Z} the set of integers
- \mathbb{Q} the set of rational numbers
- \mathbb{R} the set of real numbers
- \mathbb{C} the set of complex numbers

1. State whether the following are true, false, or not assertions.
 - (a) If (9 is even or $5 \geq 5$) then $-1^2 = 1$
 - (b) If $k \in \mathbb{N}$ then $x^2 + kx + 1 > 0$
 - (c) 0 is positive
 - (d) Let n be a non-negative integer.
 - (e) If $x \in \mathbb{R}$ then $x = \sqrt{x^2}$
2. For each assertion in Question 1 that is an implication,
 - (a) state the converse of the implication
 - (b) determine whether the converse holds
3. State the negation of each of the following statements (assuming that A , B and C are themselves statements with truth values):
 - (a) A or (B and not(C))
 - (b) (A and not B) or C
 - (c) ((not(A)) or (B)) and (C and not(D))

Definition. For integers a and b , we say that “ a divides b ” (written as “ $a \mid b$ ”) if there exists an integer q such that $b = qa$. Otherwise a does not divide b and we can write “ $a \nmid b$ ”.

4. Explain why each of the following statements is false:
 - (a) $8 \mid 28$
 - (b) $\forall n \in \mathbb{N}, 16 \mid n^2$ if and only if $8 \mid n$
 - (c) If $x, y \in \mathbb{R}$ such that $x > 0$ and $y > 0$, then $(x + 7)^2 + (y + 24)^2 \leq 25^2$
 - (d) $\forall x \in \mathbb{R}, 123x^4 > \frac{x^6}{456789}$

(over)

5. Rewrite the following statements as English sentences. Also indicate whether each statement is true or false.
- (a) $\exists x \in \mathbb{Z}$ s.t. $\exists y \in \mathbb{Q}$ s.t. $x < y$.
 - (b) $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Q}, x \leq y$.
6. Find the negation of each statement in Question 5 and indicate whether it is true or false.
7. Let $x, y \in \mathbb{Z}$. Prove that xy is odd if and only if x and y are both odd.
8. Prove: $\forall x \in \mathbb{Z}, 3 \mid (x^3 - x)$.
9. Consider the statement: $\forall x \in \mathbb{Z}, x \text{ is odd} \Rightarrow 4 \mid (7x - x^3)$.
- (a) Is this statement true or false? Justify your answer either with a proof or else with a counter-example.
 - (b) What is the negation of the statement?