

**Instructions**

- Answer each question completely; justify your answers.
  - This assignment is due at 17:00 on Wednesday March 9th in Assignment Box #44.
1. Find integers  $q$  and  $r$  with  $0 \leq r < |b|$  such that  $a = qb + r$ :
    - (a)  $a = 128, b = 5$
    - (b)  $a = -8131, b = 7$
    - (c)  $a = -9762, b = -19$
  2. Let  $a$  and  $b$  be integers that are not both zero. Prove that  $\gcd(a, a + b) = \gcd(a, b)$ .
  3. Let  $a = 7680$  and  $b = 912$ . Let  $g$  be the greatest common divisor of  $a$  and  $b$ .
    - (a) Find  $g$ .
    - (b) Find integers  $m$  and  $n$  such that  $ma + nb = g$ .
  4. Find integers  $x$  and  $y$  such that  $154x + 260y = 4$ .
  5. Show that there is no integral solution to  $196x + 245y = 3$ .
  6. Prove: if  $k \in \mathbb{N}$  then  $\gcd(3k + 2, 5k + 3) = 1$ .
  7. Find the prime decompositions for:
    - (a)  $n = 123456$
    - (b)  $n = 5050$
  8. Exercise 4.3.9.
  9. Exercise 4.3.26, part (b).
  10. Exercise 4.3.32, parts (b) and (c).