

Instructions

- Answer each question completely; justify your answers.
 - This assignment is due at 17:00 on Wednesday February 17th in Assignment Box #44.
1. Let $A = \mathbb{R}^2$ and define \preceq on A by $(a, b) \preceq (x, y)$ if $a \leq x$ and $b \geq y$.
 - (a) Show that (A, \preceq) is a poset.
 - (b) Is the poset totally ordered?
 - (c) What is the least upper bound on $(\sqrt{3}, 7)$ and $(4, -5)$?
 - (d) What is the greatest lower bound on $(\pi, \frac{5}{6})$ and $(0, -\frac{1}{2})$?
 2. Let $f : \mathbb{N} \rightarrow \mathbb{Q}$ be defined by $f(x) = \frac{x-2}{x+1}$.
 - (a) Prove or disprove: f is surjective.
 - (b) Prove or disprove: f is injective.
 - (c) Is f bijective?
 3. Let $A = \{x \in \mathbb{R} \mid x \neq -\frac{1}{2}\}$ and define $f : A \rightarrow \mathbb{R}$ by $f(x) = \frac{6x}{2x+1}$.
 - (a) Show that f is injective.
 - (b) Is f surjective?
 - (c) What is the range of f ?
 - (d) Let B be the range of f and define $g : A \rightarrow B$ such that $g : x \mapsto f(x)$. What is g^{-1} ?
 4. Exercise 3.2.10, except part (a).
 5. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions. Prove that if $g \circ f$ is injective then f is injective.
 6. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions. Prove that if $g \circ f$ is surjective and g is injective then f is surjective.
 7. Prove that $(2, 4)$ and $(-1, 17)$ have the same cardinality.
 8. Exercise 3.3.19, parts (a) and (d).
 9. Exercise 3.3.20, except parts (a) and (e).
 10. Is the following statement true or false: if A and B are sets such that $A \subsetneq B$ then $|A| < |B|$. Explain.