MATH 2320 – Discrete Mathematics **Assignment #8**
Fall 2018

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday November 22nd in Assignment Box #41.

1. Solve the following congruences:
   (a) $7x \equiv 18 \pmod{43102}$
   (b) $3x \equiv 19 \pmod{48}$

2. Solve the following systems of congruences:
   (a) $5x - 2y \equiv 0 \pmod{11}$ and $2x + y \equiv 8 \pmod{11}$
   (b) $8x + 4y \equiv 2 \pmod{23}$ and $x - 3y \equiv 7 \pmod{23}$
   (c) $6x - 7y \equiv 8 \pmod{33}$ and $4x + 2y \equiv 3 \pmod{33}$

3. Solve the following system of congruences: $\begin{aligned} x &\equiv 42 \pmod{77} \\ x &\equiv 77 \pmod{100} \end{aligned}$

4. Solve the following system of congruences: $\begin{aligned} x &\equiv 7 \pmod{12} \\ x &\equiv 5 \pmod{25} \\ x &\equiv 9 \pmod{37} \end{aligned}$

5. Solve the following system of congruences: $\begin{aligned} x &\equiv 3 \pmod{4} \\ x &\equiv 2 \pmod{9} \\ x &\equiv 4 \pmod{25} \\ x &\equiv 7 \pmod{49} \end{aligned}$

6. Prove: for each integer $n \geqslant 1$, $n^3 + 5n$ is divisible by 6.

7. Let $n \in \mathbb{N}$. Prove that $\displaystyle\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$.

8. Suppose that $a_1, a_2, a_3, \ldots$ is an arithmetic sequence with $a_1 = a$ and common difference d. Prove that the sum of the first n terms is $S_n = \dfrac{n(2a + (n-1)d)}{2}$.

9. Suppose the sum of the first eight terms of an arithmetic sequence is thrice the sum of the first five terms, and, moreover, the sum of the first ten terms is 1475. Find a formula for the sequence and calculate the sum of the first 50 terms.

10. If the second term of a geometric sequence is 8 and the seventh term is $-\frac{1}{4}$, find the sum of the first 100 terms.

11. Let $a_0 = 5$, $a_1 = 17$, and for each $n \geqslant 1$ define $a_{n+1} = -8a_n - 16a_{n-1}$. Find a formula for a_n.

12. Consider the sequence defined by $a_0 = 2$, $a_1 = 3$ and for each $n \geqslant 2$, $a_n = -a_{n-1} - a_{n-2}$. Determine a_n in general, and then use your solution to determine a_3.

(over)

13. In a group of 91 students, 73 are taking Math, 42 are taking English, and 7 are taking neither. How many students are taking both English and Math?

14. When a group of people were surveyed about whether they like the coffee at three local stores known as Alice's, Bob's and Charlie's, 626 said they liked Alice's, 452 liked Bob's, and 440 liked Charlie's. Some of these people liked more than one store's coffee: 109 liked both Alice's and Bob's, 51 liked both Alice's and Charlie's, and 311 liked both Bob's and Charlie's. If 9 people like all three coffee stores, how many people were surveyed?

15. Suppose U is a set containing 75 elements and that A_1, A_2, A_3 and A_4 are subsets of U such that each subset A_i contains 28 elements, the intersection of any two of the four subsets contains 12 elements, the intersection of any three of the four subsets contains 5 elements, and the intersection of all four subsets contains 1 element.

    (a) What is $|A_1 \cup A_2 \cup A_3 \cup A_4|$?

    (b) How many elements belong to exactly two of the four subsets?