MATH 2320 – Discrete Mathematics — Assignment #6
Fall 2018

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday November 1st in Assignment Box #41.

1. Define $f : (-1, 1) \to \mathbb{R}$ such that $f(x) = \frac{x}{1-x^2}$.
   (a) Prove that f is bijective.
   (b) Prove that there is a one-to-one correspondence between $(-1, 1)$ and $(0, 1)$.
   (c) Deduce that $\mathbb{R}$ is uncountable.

2. Prove that each of the following sets is countable by listing its elements in a systematic way with a first element, second element, etc. List at least the first ten elements of each set.
   (a) All integral powers of 2 (*i.e.*, every number of the form 2^n where n is an integer).
   (b) Those natural numbers that leave a remainder of 1 when divided by 3.
   (c) $\mathbb{N} \times \{0, 1, 2\}$

   Determine, with justification, whether each of the following sets is finite, countably infinite, or uncountable.
   (a) $\{x \in \mathbb{N} \mid 3 < x < 4\}$
   (b) $\{x \in \mathbb{Q} \mid 3 < x < 4\}$
   (c) $\{x \in \mathbb{R} \mid 3 < x < 4\}$
   (d) $\{a + bi \in \mathbb{C} \mid a, b \in \mathbb{N}\}$
   (e) $\{(a, b) \in \mathbb{Q}^2 \mid a + b = 1\}$
   (f) $\{(a, b) \in \mathbb{R}^2 \mid b = \sqrt{1 - a^2}\}$

3. Is the following statement true or false: if A and B are sets such that $A \subset B$ then $|A| < |B|$. Explain.

4. Find integers q and r with $0 \leqslant r < |b|$ such that $a = qb + r$:
   (a) $a = 139$, $b = 6$
   (b) $a = -8241$, $b = 5$
   (c) $a = -9362$, $b = -17$
   (d) $a = 27133$, $b = -45$

5. Let $a = 8850$ and $b = 975$. Let g be the greatest common divisor of a and b.
   (a) Find g.
   (b) Find integers m and n such that $ma + nb = g$.
   (c) What is the least common multiple of a and b?

6. Let a and b be integers that are not both zero. Prove that $\gcd(a, a + b) = \gcd(a, b)$.