MATH 2320 – Discrete Mathematics   Assignment #5
Fall 2018

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday October 25th in Assignment Box #41.

1. Let $S = \{1, 2, 3, 4\}$ and $T = \{a, b, c, d\}$. Define functions $f : S \to T$ and $g : S \to S$ such that $f = \{(1, d), (2, b), (3, c), (4, a)\}$ and $g = \{(1, 3), (2, 4), (3, 1), (4, 4)\}$.

    (a) Either find $f \circ f$ or explain why it does not exist.

    (b) Either find $f \circ g$ or explain why it does not exist.

    (c) Either find $g \circ f$ or explain why it does not exist.

    (d) Either find $g \circ g$ or explain why it does not exist.

    (e) Is f surjective? Is f injective? Find f^{-1} if it exists.

    (f) Is g surjective? Is g injective? Find g^{-1} if it exists.

2. Let $A = \{x \in \mathbb{R} \mid x \neq \frac{1}{3}\}$ and define $f : A \to \mathbb{R}$ by $f(x) = \frac{x}{3x-1}$.

    (a) Show that f is injective.

    (b) Is f is surjective?

    (c) What is the range of f?

    (d) Let D be the range of f and define $g : A \to D$ such that $g : x \mapsto f(x)$. What is g^{-1}?

3. Let $A = \mathbb{R} \setminus \{0, 1\}$. Define functions $f : A \to A$, $g : A \to A$ and $h : A \to A$ by $f(x) = \frac{1}{x}$, $g(x) = \frac{x-1}{x}$ and $h(x) = \frac{1}{1-x}$.

    (a) For each function:

        i. Determine whether the function is bijective.

        ii. If it is bijective, find its inverse.

    (b) What is $f \circ f$?

    (c) What is $f \circ g$?

    (d) What is $f \circ h$?

    (e) What is $f \circ g \circ h$?

4. Let $f : A \to B$ and $g : B \to C$ be functions. Prove that if $g \circ f$ is injective then f is injective.

5. Let $f : A \to B$ and $g : B \to C$ be functions. Prove that if $g \circ f$ is surjective and g is injective then f is surjective.

6. Show that there is a bijection between the sets $(-10, 5)$ and $(1, 2)$.