MATH 2320 – Discrete Mathematics Assignment #4
Fall 2018

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday October 11th in Assignment Box #41.

1. Define the relation $\sim$ on $\mathbb{Z}$ by $a \sim b$ if $(a^2 - b^2)$ is even.
   (a) Prove that $\sim$ is an equivalence relation.
   (b) What is $\overline{1}$?
   (c) What is $\mathbb{Z}/{\sim}$?
2. Let $A = \{1, 2, 3, \ldots, 15\}$ and define $\preceq$ on A by $a \preceq b$ if a divides b.
   (a) Show that $(A, \preceq)$ is a poset.
   (b) Is the poset totally ordered?
   (c) Does this poset have a maximum? If yes, what is it?
   (d) Does this poset have a minimum? If yes, what is it?
   (e) What are the maximal elements of this poset?
   (f) What are the minimal elements of this poset?
   (g) What is the least upper bound of elements 3 and 4?
   (h) What is the greatest lower bound of elements 5 and 6?
   (i) Draw the poset's Hasse diagram.
3. Let $A = \mathbb{R}^2$ and define $\preceq$ on A by $(a, b) \preceq (x, y)$ if $a \leqslant x$ and $b \geqslant y$.
   (a) Show that $(A, \preceq)$ is a poset.
   (b) Is the poset totally ordered?
   (c) What is the least upper bound on $(9, -\sqrt{3})$ and $(4, 2)$?
   (d) What is the greatest lower bound on $(\pi^2, \frac{4}{3})$ and $(-7, \frac{3}{4})$?
4. Let $A = \{1, 2, 3, \ldots, 8\}$ and define the function $g : \mathcal{P}(A) \to \mathbb{Z}$ so that $g(x) = |x|$.
   (a) What is the domain of g?
   (b) How many elements are in the domain of g?
   (c) What is the range of g?
   (d) Is g surjective?
   (e) Is g injective?
   (f) Is g bijective?
5. Define $h : \mathbb{N}^2 \to \mathbb{N}$ by $h : (x, y) \mapsto (x + y)$.
   (a) State the domain of h.
   (b) State the range of h.
   (c) Is h surjective?
   (d) Is h injective?
   (e) Is h bijective?